3D forward modelling and inversion of CSEM data at the San Nicolás massive sulphide deposit

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SUMMARY

Scalar CSAMT measurements collected over the San Nicolás massive sulphide deposit are inverted using a 3D frequency domain inversion methodology. Maxwell's equations are solved in the frequency domain using finite volumes on a staggered grid, while the inversion is solved using a Gauss-Newton methodology. A corrective-source formulation is used to reduce the large 3D numerical domain required for the CSAMT survey configuration. The resulting conductivity is compared with a 1D interpretation; the 3D inversion model shows an improved recovery of the deposit geometry. A comparison with a conductivity model obtained using time-domain measurements and a similar inversion methodology show a great agreement in the resulting models despite large differences in the survey configurations.
Keywords: finite volume, Gauss-Newton, mining

INTRODUCTION

The San Nicolás massive sulphide deposit in central Mexico has been host to a vast array of geophysical and geological exploration: gravity, magnetics, airborne EM, ground-based time domain EM, ground-based frequency domain EM and extensive drilling. The result is a very detailed 3D geological model of the main deposit body and overburden (Philips, Oldenburg, Chen, Li, & Routh, 2001). The large amount of prior information makes the San Nicolás deposit an ideal candidate to evaluate 3D forward modelling and inversion of electromagnetic measurements. Three lines of conventional scalar controlled source audio magnetotelluric (CSAMT) measurements were collected over the San Nicolás massive sulphide deposit with a single grounded source situated several kilometers from the deposit. Interpretation of the measurements was previously limited to 1D inversion of each sounding. Here we invert the measurements in 3D.

FINITE-VOLUME FORWARD MODELLING

Maxwell's equations in the frequency domain can be written as

$$\begin{aligned} \nabla\times\,\mathbf{E} - \imath\omega\mu\mathbf{H} &= 0 && (1a)\\ \nabla\times\,\mathbf{H} - (\sigma - \imath\omega\epsilon)\mathbf{E} &= \mathbf{s}_r(\omega), && (1b)\end{aligned}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields, μ is the permeability, σ is the electrical conductivity, ϵ is the permittivity and $\mathbf{s}_r(\omega)$ is the frequency-dependent source current density. The 3D forward problem over a finite discrete domain Ω is formulated using a finite-volume formulation on a staggered grid using a Helmholtz decomposition such that $\mathbf{E} = \mathbf{A} + \nabla\phi$ (Haber, Ascher, Aruliah, & Oldenburg, 2000), (Haber & Ascher, 2001). Boundary conditions of $n\times\mathbf{H} = 0$ are specified on $\partial\Omega$. Defining $\hat{\sigma} = \sigma - i\omega\epsilon$, the resulting discrete system of equations is

$$\begin{pmatrix} L_\mu + i\omega M_{\hat{\sigma}} & i\omega M_{\hat{\sigma}}\nabla_h \\ \nabla_h\cdot\, M_{\hat{\sigma}} & \nabla_h\cdot\, M_{\hat{\sigma}}\nabla_h \end{pmatrix}\begin{pmatrix}\mathbf{A}\\ \phi\end{pmatrix} = \begin{pmatrix} i\omega\mathbf{s}_r \\ \nabla_h\cdot\,\mathbf{s}_r\end{pmatrix} \quad (2)$$

where $\nabla_h\cdot$, $\nabla_h\times$ and ∇_h are matrices arising from the discretization of the corresponding continuous operators, $M_{\hat{\sigma}}$ arises from the operator $\hat{\sigma}(\cdot)$ and L_μ is the discretization of the operator $\nabla\times\,\mu^{-1}\nabla\times\, - \nabla\mu^{-1}\nabla\cdot$.

We write the discrete forward system as a generic linear system of equations, $A(\mathbf{m})\mathbf{u} = \mathbf{q}$ where $A(\mathbf{m})$ is a sparse discretization of Maxwell's equations, $\mathbf{u} = [\mathbf{A}, \phi]^T$ contains the vector and scalar potentials and $\mathbf{q}$ is the discretized source term. The predicted data are then written as $\mathbf{d}^{pred} = Q\mathbf{u}$ where Q is a projection matrix that maps the resulting potentials to a field component of the data. The large, sparse linear system of the forward equations is solved using a BiCGSTAB conjugate gradient solver using an iLU pre-conditioner.

Domain reduction

For a numerical solution to equation 2, the volume Ω must be large enough to contain the source $\mathbf{s}_r$ and be of sufficient extent that the boundary conditions are satisfied. A CSAMT survey configuration with the source several kilometers from the receivers results in a large numerical domain and the resulting forward and inverse problem can become prohibitively large. To reduce the domain of interest we introduce a correction procedure which has many elements of a primary/secondary field formulation.

To begin, we generate a reference solution to Maxwell's equations, $\mathbf{E}_p$, $\mathbf{H}_p$ in Ω for a conductivity model σ_p. We form a smaller domain around the area of interest $\Omega_s \subset \Omega$, such that the source $\mathbf{s}_r$ is outside Ω_s. The linear system equation 2 is formed on the domain Ω_s for the conductivity model σ_p. The solution of these fields is then compared to the reference fields sampled at the discrete locations. The discrepancy is used as a corrective source. The consequence is that the solution to equation 2 on Ω_s matches the reference fields. The corrective source on the small domain can be evaluated as

$$\mathbf{s}_r^s = \nabla \times \, _h\mathbf{H}_p - \sigma_p \mathbf{E}_p \tag{3}$$

$\mathbf{s}_r^s$ is then used in place of $\mathbf{s}_r$ in equation 2 to solve for the total potential $\mathbf{u}$ on Ω_s for a given conductivity model.

INVERSION

In the inverse problem, we attempt to find a conductivity model that matches the observed fields to within a specified misfit while minimizing some measure of the models size. To accomplish this, we minimize an objective function

$$\min \frac{1}{2}\|W_d(Q\mathbf{u} - \mathbf{d}^{obs})\|^2 + \frac{1}{2}\beta\|W_m(\mathbf{m} - \mathbf{m}_{ref}\|^2 \tag{4}$$

where $\mathbf{d}^{obs}$ are the observed electric and magnetic fields, represented as real and imaginary values or amplitude and phases of electric and magnetic fields or the ratios of the fields (impedance); W_d is a diagonal matrix containing the standard deviation of the measurements and W_m is a model weighting matrix. The solution to the inverse problem is similar to (Haber, Ascher, & Oldenburg, 2004). A Gauss-Newton approach is used to find the minimizer of equation 4, in which a model perturbation is obtained through

$$(J^T W_d^T W_d J + \beta W_m^T W_m)\mathbf{p} = -\mathbf{g}(\mathbf{m}) \tag{5}$$

where $\mathbf{g}(\mathbf{m})$ is the gradient of the objective function and J is the sensitivity matrix. In the 3D electromagnetic inversion problem, it is computationally prohibitive to explicitly compute and store the sensitivity matrix. However, the product of the sensitivity times a vector can be efficiently obtained through $J = -Q\mathbf{A}(\mathbf{m})^{-1}G$, where the matrix $G = \partial[\mathbf{A}(\mathbf{m})\mathbf{u}]/\partial\mathbf{m}$. The step $\mathbf{p}$ in equation 5 is solved iteratively using a pre-conditioned conjugate gradient solver. Updates to the model are obtained through $\mathbf{m}_{n+1} = \mathbf{m}_n + \alpha\mathbf{p}$ where the step-length parameter α is chosen through a polynomial line search such that the new model $\mathbf{m}_{n+1}$ adequately reduces the objective function (Kelley, 1999). The tradeoff parameter β is reduced through a cooling schedule until the target misfit is achieved. An iterative-Tihkonov regularization is used in which the reference model is changed during each change to the regularization parameter such that $m_{ref} = m_n$, so that the smallest model term in the regularization is $W_s(m_{n+1} - m_n)$.

Inversion workflow

Interpreting EM data using 3D inversions is challenging, and successful application requires several stages that must be done prior to moving onto the next. The implementation of an efficient workflow requires an informed and skilled geophysicist.

The first step in working with EM data is to fully understand the nature of the measurements and their units. Confusion even arises because of differences in the definitions of the x, y, z coordinate systems in the field and the processing codes. Although the step of understanding the data should be trivial, details of the data collection, normalization and processing are often dropped as the data are passed along, or details are poorly documented. In practise we have often spent far more time on this step than we have in carrying out the inversion. We encourage companies and contractors to address this issue.

Forward modelling is a critical component in the inversion of EM data and we must be able to model the fields quickly and accurately. In order to perform a forward modelling it is required to create a background model and a spatial discretization. It may be necessary to revise the background model or the discretization as interpretation of the measurements proceeds.

Prescribing uncertainties to the data is difficult yet critical step in the inversion process. The final assignment is problem dependent and may involve determining what scale of features of the data should be reproduced, information from repeat observations, reciprocity checks, estimation of errors in the forward modelling, and interrogating misfit maps between the observed and predicted data.

At preliminary stages we want to solve smaller problems so that turn-around on the inversion is fairly quick. It is recommended to invert frequencies individually and evaluate the results in terms of the misfit the resulting model. In the inversions to produce a final model for interpretation, the discretization can be made smaller.

CSEM measurements at the San Nicolás deposit, Mexico

The San Nicolás deposit is a Cu-Zn massive sulphide located in the state of Zacatecas in central Mexico. The main sulphide deposit is a continuous but geometrically complex body of sulphides which is covered by 175-250m of variable composition overburden (figure 1). The local geology is also complex and contains numerous sedimentary and volcanic units. The sulphide deposit presents an electrical conductivity contrast with most of the geologic units in the area, however some of the overburden, the tertiary volcanic breccia, has a conductivity in the range of that found in the sulphide. The thick conductive overburden, and relatively deep massive sulphide body provide a challenging scenario for electrical geophysical techniques.

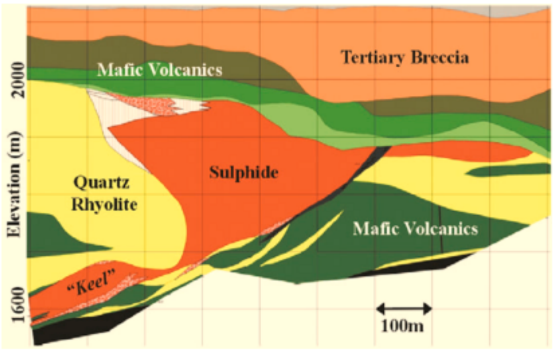

Figure 1: San Nicolás massive sulphide geological cross section.

Conventional scalar CSAMT measurements were made over the San Nicolás deposit with the survey geometry shown in figure 3. Three 1.5 km survey lines were collected over the deposit. Each line contains 60 stations, spaced 25m apart, with lines separated by 200 m. The CSAMT measurements were collected from a single transmitter, located 3.5 kilometers from the receivers. The regional background resistivity is about 100 Ohm-m, making measurements at frequencies below 128 Hz in the near-field or transition zone.

The data consist of scalar impedances, ratios of the measured electric and magnetic fields, collected at 15 frequencies between 0.5 and 8192 Hz. A 1D inversion using all available frequencies was performed using a 1D CSEM code (Routh & Oldenburg, 1999). The resulting conductivity model was able to image the sulphide conductor at depth (Philips et al., 2001).

3D inversion results

For the 3D inversion we used the amplitude and phase of impedance measurements at four frequencies, 0.5, 8, 64, 256 Hz. The result is shown in figure 6a. The inversion was started from a starting model of a 100 Ohm-m halfspace, and converged to the target misfit in 7 iterations. The solution to the forward problem was computed in parallel, using 1 processor per frequency, with the inversion taking a total of 40 hours to run.

The same standard deviations used in the 1D inversions were used in the 3D inversion, that is 5% of the amplitude, plus a small floor, and 2 degrees error on the phase. The observed and predicted data for 8 Hz are shown in figure 4.

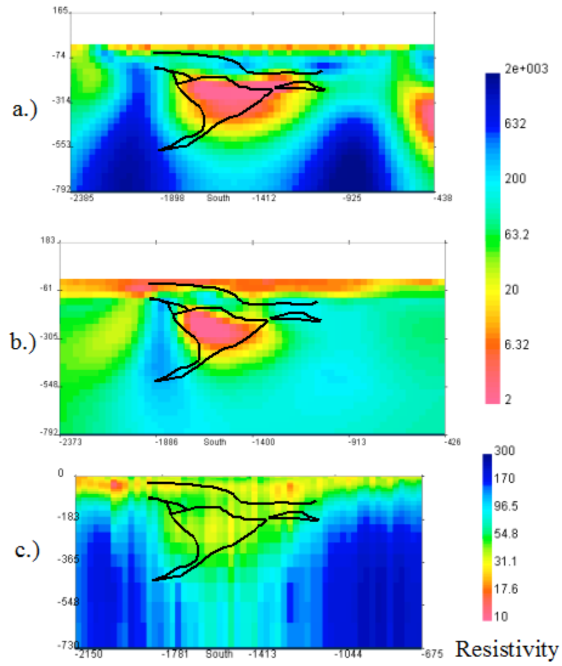

Figure 2: San Nicolás conductivity models: a.) 3D CSEM, b.) 3D time domain EM, c.) stitched 1D CSEM.

The resulting conductivity model recovered from the 3D CSAMT inversion shows an excellent correspondence with the outline of the main massive sulphide body delineated through drilling. The resulting model also picks up some of the conductive overburden and the resistive volcanic units between the overburden and the deposit.

In addition to the CSEM data, time domain UTEM measurements were also collected over the San Nicolás deposit, and interpreted using a similar 3D methodology (Napier, Oldenburg, Haber, & Shekhtman, 2006), with the resulting inversion shown in figure 6b. The conductivity models in figures 6a and 6b are shown on the same color scale, and show very similar structure. The stitched conductivity model obtained through a 1D inversion is shown in figure 6c. The 1D model shows the conductive sulphide body at depth, and picks up the conductive overburden layer. However, the resulting image of the deposit is not as clear as in the 3D inversion models.

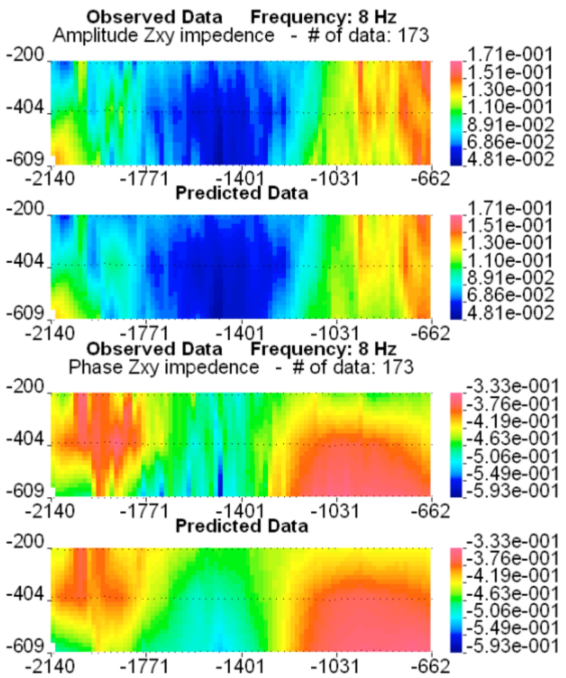

Figure 3: Observed and predicted amplitude and phase impedance measurements for 8 Hz

Conclusions

The CSEM data collected at San Nicolás are from a single transmitter, and a relatively large spacing between the survey lines, the CSAMT dataset over the San Nicolás deposit is relatively limited dataset when compared to other electromagnetic datasets used in 3D inversions (Napier et al., 2006), (Oldenburg, Eso, Napier, & Haber, 2005). However, the resulting 3D interpretation of the CSAMT model shows a more defined deposit model when compared with the 1D interpretation and is in agreement with an independent 3D EM interpretation. The 1D models show a sulphide resistivity of 20-30 Ohm-m for the main body, while the 3D models indicate a more conductive sulphide, with resistivities in the range of 2-10 Ohm-m.

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