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Theoretical development of the differential scattering decomposition for the 3D resistivity experiment

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ABSTRACT

In any numerical solution of the DC resistivity experiment, care must be taken to deal with strong heterogeneity of electrical conductivity. In order to examine the importance of conductivity contrasts, we develop a scattering decomposition of the DC resistivity equation in the sparse differential domain as opposed to the traditional dense integral formulation of scattering-type equations. We remove the singularity in the differential scattered series via separation of primary and secondary conductivity, thereby avoiding the need to address the singularity in a Green's function. The differential scattering series is observed to diverge for large conductivity contrasts and to converge for small contrasts. We derive a convergence criterion, in terms of matrix norms for the weak-form finite-volume equations, that accounts for both the magnitude and distribution of heterogeneity of electrical conductivity. We demonstrate the relationship between the differential scattering series and the Fréchet derivative of the electrical potential with respect to electrical conductivity, and we show how the development may be applied to the inverse problem. For linearization associated with the Fréchet derivative to be valid, the perturbation in electrical conductivity must be small as defined by the convergence of the scattered series. The differential scattering formulation also provides an efficient tool for gaining insight into charge accumulation across contrasts in electrical conductivity, and we present a derivation that equates accumulated surface charge density to the source of scattered potential.

INTRODUCTION

The solution of the low-frequency (DC) resistivity equation for the electrical potential distribution, given a general electrical conductivity field, can be approached from a variety of directions. To allow for the discretization of arbitrary three-dimensional (3D) earth models, a finite-difference or finite-element approximation of the governing differential equation is usually used (Li and Spitzer 2002). Whichever numerical method is chosen for solving the differential equation, care must be taken to deal with strong heterogeneity of electrical conductivity. For example, Spitzer and Wurmstich (1999) provided a review of 3D finite-difference techniques in the context of the DC resistivity problem and they demonstrated that the main discrepancies in numerical results stem from the different approaches used to handle discontinuities in electrical conductivity.

We investigate the role of contrasts in electrical conductivity by describing the behaviour of the potential solution using a scattering decomposition. The development is unique in that we formulate the scattered series in the differential domain as opposed to the traditional integral formulation of scattering-type equations. This is significant in that large 3D problems with general electrical conductivity distributions are almost invariably solved using a numerical approximation of the differential equation, not the integral equation.

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