

The point-spread function measure of resolution for the 3-D electrical resistivity experiment

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SUMMARY

The solution appraisal component of the inverse problem involves investigation of the relationship between our estimated model and the actual model. However, full appraisal is difficult for large 3-D problems such as electrical resistivity tomography (ERT). We tackle the appraisal problem for 3-D ERT via the point-spread functions (PSFs) of the linearized resolution matrix. The PSFs represent the impulse response of the inverse solution and quantify our parameter-specific resolving capability. We implement an iterative least-squares solution of the PSF for the ERT experiment, using on-the-fly calculation of the sensitivity via an adjoint integral equation with stored Green's functions and subgrid reduction. For a synthetic example, analysis of individual PSFs demonstrates the truly 3-D character of the resolution. The PSFs for the ERT experiment are Gaussian-like in shape, with directional asymmetry and significant off-diagonal features. Computation of attributes representative of the blurring and localization of the PSF reveal significant spatial dependence of the resolution with some correlation to the electrode infrastructure. Application to a time-lapse ground-water monitoring experiment demonstrates the utility of the PSF for assessing feature discrimination, predicting artefacts and identifying model dependence of resolution. For a judicious selection of model parameters, we analyse the PSFs and their attributes to quantify the case-specific localized resolving capability and its variability over regions of interest. We observe approximate interborehole resolving capability of less than 1–1.5 m in the vertical direction and less than 1–2.5 m in the horizontal direction. Resolving capability deteriorates significantly outside the electrode infrastructure.

Key words: Inverse theory; Tomography; Electrical properties; Hydrogeophysics; Hydrology.

1 INTRODUCTION

The crux of any inverse problem is often considered to be the estimation component, whereby we construct a model estimate to fit our data. However, the credibility of geophysical inversion or tomography suffers from a lack of quantification of the reliability of our model estimates. Although we often cannot establish the true accuracy of our model estimate, the tools of solution appraisal allow for an investigation of the relationship between our estimated model and the actual model.

For small linear problems, solution appraisal is relatively straightforward (e.g. Menke 1989). This is not so for non-linear problems, since no formal theory exists to deal with solution appraisal (Oldenburg 1983; Snieder 1998). In addition, for large 3-D problems with multiscale model domains, the computational burden of solution appraisal can be prohibitive (Nolet *et al.* 1999). The 3-D electrical resistivity tomography (ERT) problem is both non-linear and large with several thousands of data and several hundreds of thousands of model parameters.

Furthermore, the full appraisal problem has both an error propagation aspect and a resolution aspect. Since data noise is an implicit component of the resolving capability, we focus only on the model resolution (Backus & Gilbert 1968; Menke 1989). Since the ERT problem is non-linear, we employ the linearized form of the resolution. Ultimately, it is unlikely that geophysical inversion will ever exist in absence of an interpreter or decision maker, (Scales & Snieder 2000) and when faced with geophysical

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