

# Time-Frequency Filtering using Fractional Spline Wavelet Packets

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## Abstract

Here we present a method for time-frequency filtering using Fractional Spline Wavelet Packets. Possible applications are in improved localization of reflections and ground roll elimination.

## Introduction

The standard filters used to eliminate undesirable components from seismic traces are frequency filters and velocity filters. The problem is that the reflections - typically the broad band component of the signal - tend to be affected by filtering operations, which suppress frequencies present in the noise, without regard to time-space position.

To remedy this difficulty a number of time-frequency decompositions have been attempted such as the spectrogram, the Gabor atomic decomposition, the wavelet transforms, continuous and discrete, and the Wigner-Ville transforms [Mallat, Steeghs]. All these decompositions provide systematic means to perform time-frequency filtering, a general term for locally manipulating the frequency content of a signal.

Key aspects of the decompositions are their ability to sparsely represent data, and their ability to be data adaptive. By using non-linear methods it is possible to find an optimal representation for a particular signal from a redundant dictionary of basis functions. We use a combination of two such methods: the Coiffmann-Wirkerhouser Best Basis (BB) selection and Matching Pursuit (MP).

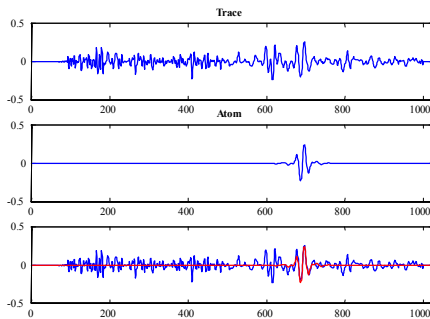


Figure 1 - The first trace is shown with the predominant atom which is parametrized at point 700 by a Fractional Spline Wavelet with  $\alpha = 3.90$ .

The dictionary we use is composed of Fractional Spline Wavelet Packets (FSWP) [Herrmann1]. These packets are derived from the Fractional Spline Wavelets [Unser]. They have all properties needed to perform atomic decompositions and fit our purposes mainly because they can continuously vary their time-frequency characteristics, i.e. their decay rate for high frequencies (regularity) and their behavior at the origin (vanishing moments and fractional order of differentiation). In this way we can construct fractional splines that are  $\alpha$ -Hölder regular, (with  $\alpha \in (-1/2, \infty)$ ) at the knots. Consequently, our atoms can be extremely smooth, almost sine waves or sinc func-

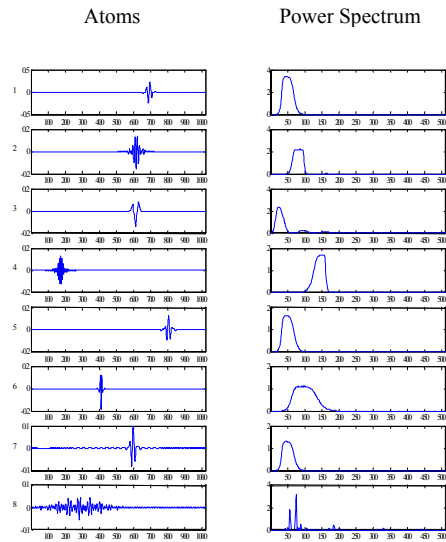


Figure 2 – The eight more representative atoms in the decomposition of the trace.

tions, or jumps. Using a well founded computational scheme, these Fractional Spline Wavelet Packets allow us to find waveforms from a large dictionary that best fit events in the data. In this paper we will show that this FSWP decomposition may perform particularly well for seismic data processing, such as the removal of the ground roll noise component.

## Methodology

To reduce the computational complexity we construct our MP dictionary by first conducting a Best Bases selection on the FSWP. So for each trace we compute several  $\alpha$ -indexed wavelet packet trees. Then, for each tree we will use Coifman-Wirkerhouser Algorithm to find the BB representation, followed by the

## Fractional Spline Wavelet Packets

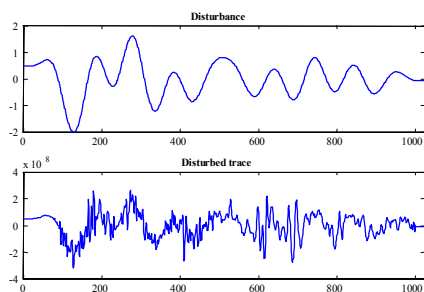


Figure 3 – A low frequency disturbance and the disturbed trace.

“greedy” selection of atoms, using the MP algorithm. The output is an ordered list of atoms, from the most predominant to the faintest. Via composition we can reconstruct the trace again from the list of atoms. This reconstruction converges to the original trace, as the number of atoms tends to infinity. When the dictionary sufficiently closely matches events in the data, the atomic decomposition, followed by a reconstruction using atoms that exceed a noise dependent threshold, also removes additive random noise [Mallat]. In practice, we obtain almost perfect reconstruction for a number of atoms far less (typically 10 % of the sample number) than the number of samples in the trace.

### Example

Besides incoherent random noise removal our atomic decomposition also lends itself to eliminate large coherent noise components from the data. For this purpose we disturb the seismic trace of Figure 1, see Figure 3. The seismic trace has a length of 1024. The trace is decomposed into the 100 best correlating atoms from the dictionary. The dictionary of Best Bases vectors was formed using FSWP's with 100 equally spaced values of  $\alpha \in [-0.49, 7]$ . Figure 1 shows the most important atom in this decomposition and we see that it fits the event at position 700 quite well.

In Figure 2 we picture the eight most important atoms together with their Fourier spectrum. The estimated  $\alpha$ -indexes characterize the transition sharpness and carry information on the lithology. Refer to [Herrmann1, Herrmann2] for a discussion on the litho-stratigraphical information content of the atom's parameterization.

What we expect of our decomposition is that it fits the perceptible changes in the trace since, typically, reflections that are of most interest occur at sharp

transitions in the earth's elastic properties. Other, less impulsive components, such as the low frequency disturbance in Figure 3, are not of interest. So, what we would like to do is to remove this disturbance without affecting the broad band time-frequency characteristics of reflection events. These events carry useful information on the geology [Steeghs, Herrmann1, Herrmann2].

The Fourier spectrum of the atoms gives information about their central frequency and bandwidth. The broad band atoms are candidates to represent singularities, that is to say, reflections. The narrow band atoms can be associated with discrete frequency components of slow decay compared with the duration of the trace.

This separation can be used to filter coherent noises that usually decay slowly compared with the reflections. In our application of ground roll removal, the noise has significantly less bandwidth than the reflections. This difference can be used to separate the noise from the data. As an example we apply our method to the disturbed trace of Figure 3. First, we perform an atomic decomposition as in the previous undisturbed case. The only difference is that we use more atoms (130), to compensate for elimination of atoms that are related to the coherent noise. The eight most relevant atoms are shown in Figure 4. As we can see a number of the atoms are related to the disturbance. However, one can also see that most of the atoms pertaining to the unperturbed signal are still there. By using a simple criterion: any atom decaying slowly in a distance of half the disturbance length (which in this case is the total length) was considered as a disturbance component and not taken into account in the reconstruction. In this way we are able to accurately reconstruct the original signal. The filtered result is shown in figure 5, and can be compared with the original trace.

As we can see from these results, application of this method to the elimination of ground roll seems to be promising. As compared to common ground roll removal based on high-pass filtering, our method has the advantage that it affects the low frequencies selectively in time. Primary reason for the better performance is the data adaptation yielding an improved time-frequency localization. This localization allows us to separate the very coherent reflection events from the less coherent ground roll. It will be interesting to compare our proposed method to approaches involving wavelet decompositions mixed with velocity analysis [Deighan, Tavares].

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### Conclusion

The atomic decomposition scheme we presented in this paper excels in separating broad band from narrow band or even discrete frequency components. The data adaption in conjunction with the appropriateness of our FSWP dictionary for seismic data are responsible for the accomplished time-frequency localization. As a consequence we are able to identify atoms that are related to the data and filter out those that are related to the noise. The main advantages of our method are:

- Only amplitudes corresponding to disturbances are eliminated in a given portion of the signal.
- Singularities (points characterizing reflections) are unaffected.
- Resulting filtered signal is compressed.
- The method can be integrated with other applications of time-frequency analysis.

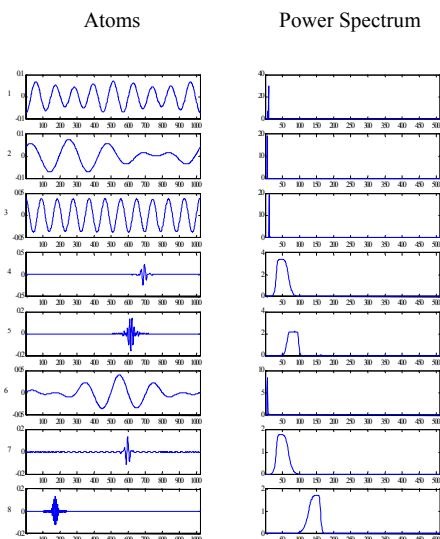


Figure 4 – The eight more important atoms in the decomposition of the disturbed trace.

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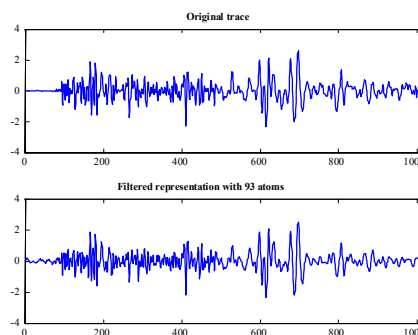


Figure 5 – The original trace and the filtered representation with 93 atoms.

### Acknowledgments

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