N 016 SCALING AND SEISMIC REFLECTIVITY: IMPLICATIONS OF SCALING ON AVO

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Abstract

AVO analysis of seismic data is based on the assumption that transitions in the earth consist of jump discontinuities only. Generalization of these transitions to more realistic transitions shows a drastic change in observed AVO behavior, especially for the large angles currently attained by increasing cable lengths. We propose a simple approach that accounts for this anomalous behavior by renormalizing the observed AVO. This approach allows for a separation of the observed AVO effects in terms of a conventional Zoeppritz contribution and a scaling contribution, when the transitions can no longer be considered as isolated jump discontinuities. After renormalization, the inverted fluctuations regain their relative magnitudes which, due to the scaling, may have been significantly distorted.

Introduction

Observed amplitude-variation-with-offset (AVO) behavior of surface seismic reflection data has led to inferences ranging from the successful identification of hydro-carbon traps to the misidentification of AVO anomalies. This mixed success may be due to oversimplification of the current transition models, which are mostly limited to jump discontinuities in the elastic properties.

Multiscale scale analysis [2, 1] on well data have shown evidence of a much broader class of apparent transitions of which the prevailing jump discontinuity, is one particular case [2, 4, 3]. As a consequence, anomalous AVO-behavior will be observed in cases where the transitions substantially deviate from the jump discontinuity. In that situation, AVO contains an additional contribution, caused by the non-trivial scale dependence of generalized transitions [2, 4, 3], which becomes apparent as the vertical wavelength increases with increasing off-set.

Wapenaar et al., 1999 propose a filter that removes the scaling contribution by keeping the vertical wavelength constant as a function of off-set. This removal, however, goes at the expense of resolution – the vertical wavelength is set to the value of the largest off-set. In our approach we aim to (i) correct for the scaling contribution without loosing resolution; (ii) to reconstruct the sharpness (scaling) of the transtions. In this way we are not only able to correctly interpret tunning effects [6, 8] but we also gain information on the reflector sharpness.

The setup of this paper is as follows. First, we briefly introduce expressions for the linearized imaged reflectivity and their relation with the continuous wavelet transform. Then we introduce a renormalization for the observed AVO, followed by a reconstruction of the transition sharpness.

Imaged seismic reflectivity

Following [8, 3] we can write, for a lateral homogeneous medium, the acoustic imaged reflectivity as follows:

$$\langle R(\mathrm{p}, z)\rangle = \frac{2\bar{q}(\mathrm{p})}{\pi}\Re\int_0^\infty \hat{p}(\mathrm{p}, z; \omega)\mathrm{d}\omega. \tag{1}$$

In this expression the imaged reflectivity equals the downward continued pressure wavefield evaluated at time zero, $\langle R(\mathrm{p}, z)\rangle \triangleq p(\mathrm{p}, z; t = 0)$ with $p(\mathrm{p}, z; t)$ the inverse temporal Fourier transformed downward continued pressure at depth-level z and ray-parameter p. $\bar{q}(\mathrm{p})$ represents the slowness defined by the averaged compressional velocity $\bar{c}$. Eq. 1 contains the source contribution and the angular brackets $\langle\,\rangle$ are used to denote bandwidth limitation.

By neglecting multiple scattering, Eq. 1 can be written [8, 3] as a spatial convolution

$$\langle R(\mathrm{p}, z)\rangle = \frac{2\bar{q}(\mathrm{p})}{\pi}(r_+(\mathrm{p}, \cdot) *_z \varphi_z)(\mathrm{p}, z). \tag{2}$$

The source function and reflection density are given by

$$\varphi_z(\mathrm{p}, z) = \varphi(\cdot 2\bar{q}(\mathrm{p}))(z) \quad \text{and} \quad r_+(\mathrm{p}, \cdot) = H(z - \cdot)r(\mathrm{p}, \cdot), \tag{3}$$

with $H(\cdot)$ the Heaviside distribution. Following [7], the reflection density can be linearized in the normalized acoustic impedance ($\Delta Z(z)$) and compressional wavespeed, Δc) fluctuations, yielding,

$$r(\mathrm{p}, z) \approx \begin{bmatrix}\frac{1}{2} & \frac{1}{2}\frac{\bar{c}^2\mathrm{p}^2}{\cos\theta(\mathrm{p})}\end{bmatrix}\partial_z \begin{bmatrix}\Delta Z & \Delta c\end{bmatrix}^T = \bar{\mathbf{M}}(\mathrm{p})\partial_z\boldsymbol{\Delta}(z). \tag{4}$$

Here, $\cos\bar{\theta}(\mathrm{p}) = \sqrt{1-\bar{c}^2\mathrm{p}^2}$ and $\Delta Z \approx \Delta c + \Delta\rho$. This expression is linear in the normalized medium fluctuations, which are assumed to be small and given by $\Delta f(z) = \frac{f(z)-\bar{f}}{\bar{f}} \ll 1$ with $f(z)$ and $\bar{f}(z)$ being either the actual/background acoustic impedance or the actual/background wavespeeds. Notice that the p-dependent factor of Eq. 4 remains non-linear in the background velocity, $\bar{c}$. Substitution of Eq. 4 into Eq. 2 finally yields

$$\langle R\rangle(\mathrm{p},z) = \frac{2\bar{q}(\mathrm{p})}{\pi}\bar{\mathbf{M}}(\mathrm{p})(\partial_z\mathbf{\Delta}_+ *_z \varphi_z)(z). \tag{5}$$

Imaged seismic reflectivity as the continuous wavelet transform

Eq. 5 can be recognized as a renormalized form of the continuous wavelet transform:

$$\langle R(\mathrm{p},z)\rangle = \frac{\bar{\mathbf{M}}(\mathrm{p})}{\sigma}\mathcal{W}\{\mathbf{\Delta}_+,\psi\}(\frac{\sigma}{2\bar{q}(p)},z), \tag{6}$$

where the wavelet transform [5] itself is given by

$$\mathcal{W}\{f,\psi\}(\sigma,z) \triangleq \sigma\frac{\mathrm{d}}{\mathrm{d}z}(f*\phi_\sigma)(z) = (f*\psi_\sigma)(z), \tag{7}$$

with ψ_σ the scale-indexed wavelets defined as the derivative of dilated, real and symmetric smoothing functions, $\phi_\sigma(z)$, with a width proportional to the scale σ. The scale σ can be interpreted as the reciprocal of the wavenumber.

Generalized reflector model

To leading order seismic waves are sensitive to singularities in the medium properties. When significant variations occur over length scales of the order of the seismic wavelength, waves are reflected. Correspondingly, wavelet coefficients are large at the location of singularities. Thus far, in seismology zero-order discontinuities are mostly used to represent transitional regions in the earth's properties. Given the singularity detection and characterization capabilities of wavelets, why limit ourselves to these singularities? Why not consider a wider class of singularities [1, 2, 4] defined by

$$\chi_+^\alpha(z) \triangleq \begin{cases} 0 & z\leq 0 \\ \frac{+z^\alpha}{\Gamma(\alpha+1)} & z>0, \end{cases} \quad \text{and} \quad \chi_-^\alpha(z) \triangleq \begin{cases} \frac{-z^\alpha}{\Gamma(\alpha+1)} & z\leq 0 \\ 0 & z>0. \end{cases} \tag{8}$$

These singularities can be used to define the medium variations as follows

$$f(z) = \bar{f}(z)[1+\Delta f(z)] \quad \text{where} \quad \Delta f(z) = c_-\chi_-^\alpha(z-z_0) + c_+\chi_+^\alpha(z-z_0). \tag{9}$$

Reflectors given by transitions defined by $\chi_-^\alpha(z)$ and $\chi_+^\alpha(z)$ are parametrized by the singularity-order exponent $\alpha \in R$, which determines the scaling properties and transitions sharpness, and by $c_\pm$, determining the transition magnitude. For $\alpha \geq 1$ the onset functions are continuous and differentiable; for $0<\alpha<1$ they are continuous but non-differentiable; and for $-1<\alpha\leq 0$ they are discontinuous and non-differentiable. Transitions, as defined in Eq. 9, yield the following behavior for the wavelet coefficients

$$|\mathcal{W}\{f,\psi\}(\sigma,z)| \leq C\sigma^\alpha \quad \text{as} \quad \sigma\to 0. \tag{10}$$

Scaling and AVO

For medium transitions no longer given by jump discontinuities ($\alpha \neq 0$), we find the AVO-behavior:

$$\langle R(\mathrm{p},z)\rangle = \overbrace{\frac{\bar{\mathbf{M}}(\mathrm{p})}{\sigma}}^{\text{intrinsic AVP}} \underbrace{\mathcal{W}\{\mathbf{\Delta}_+,\psi\}(\frac{\sigma}{2\bar{q}(p)},z)}_{\text{scaling AVP}}, \tag{11}$$

to contain two separate contributions

- **intrinsic AVP** given by the linearized Zoeppritz equations (cf. Eq. 4).
- **scaling AVP**, in case of a generalized transition, given by the scaling of the wavelet coefficients (cf. Eq. 10). For a fixed source wavelet (σ = constant), the scaling becomes (cf. Eq. 6)

$$\langle R\rangle(\mathrm{p},z) \propto \bar{\mathbf{M}}(\mathrm{p})\cos^{-\alpha}\bar{\theta}(\mathrm{p}) \quad \text{as} \quad \mathrm{p}\to 0. \tag{12}$$

Clearly, the second contribution may leed to erroneous interpretations in case the medium yields a behavior that, at the seismic resolution, mimicks the behavior of the generalized transitions as defined in Eq. 8. For example, in the case of a thin layer we will find $\alpha=-1$ (a thin layer acts as an approximation to the delta distribution), yielding a significant scaling contribution which may lead to erroneous interpretation and Zoeppritz inversion results.

Scaling correction and reconstruction

In [4, 3] it is shown that the scale exponent α can be estimated from post-stack migrated data. Given information on α for the major reflectors, we are able to (i) correct for the scaling contribution, yielding accurate Zoeppritz inversions for the magnitude of the medium variations across the transition; (ii) reconstruct the sharpness of the transition. The general procedure is as follows

- Use depth/time post-stack migrated data to estimate the α's and directions (the $+$ or $-$ in Eq. 8) of the major reflectors using the monoscale analysis method [4, 3].
- Compute the AVP-amplitudes by picking the amplitudes at the local maxima of pre-stack $(\tau - \mathrm{p})$-data.
- Correct for each reflector (local maximum) the scaling contribution by

$$\text{AVP-amplitude} \mapsto \cos\theta(\mathrm{p})^{\alpha} \times \text{AVP-amplitude}. \tag{13}$$

- Conduct the Zoeppritz inversion [7] on the renormalized amplitudes yielding estimates for the fluctuations in the acoustic impedance and wavespeed.
- Reconstruct the nature of the transitions [4, 3] using

$$f(z) = \int \sum_{i=1}^{N} \Delta f^i \chi_{\pm}^{\alpha_i - 1}(z - z_i)\mathrm{d}z, \tag{14}$$

  where the $\chi_{\pm}^{\alpha_i-1}$'s are the right-handed $(+)$ or left-handed $(-)$ transitions, defined in Eq.8. The α_i's and z_i's are the estimated exponents and location of the singularities and the Δf_i are the magnitudes of the transitions in acoustic impedance and velocity.

Example: single interface model with varying α

Fig. 1 contains examples of the "Zoeppritz reconstruction" from picked pre-stack synthetic migration amplitudes. The pre-stack migration data are obtained by migrating $(\tau - \mathrm{p})$-domain shot records generated by a layercode program ran on a series of media containing a single isolated singularity with α's given by the top plots. The transitions are normalized to yield the same increase in value across the transition for the compressional wavespeed and density. The order of the transition varies roughly between the behavior of a smooth first-order discontinuity to that of a delta function, $\alpha \in [-1, 1]$.

As the α decreases, the reconstructed compressional wavespeed (Fig. 1 top-left) increasingly deviates from the expected value. After the scaling correction (Fig. 1 top-right), deviations in the magnitudes of the velocity variations are removed. Finally, we can see that the nature of the transitions is fully restored when using the sharpness information (Fig. 1 bottom plot).

Conclusions

In this paper, a simple but useful method has been introduced which corrects Zoeppritz AVO inversion for the effects induced by scaling. These scaling effects arise when at the seismic scale the reflector does not behave as a jump discontinuity. Examples are tuning effects or differences in the characteristics of lithological boundaries.

Information on scaling is obtained using a monoscale analysis technique which provides sharpness estimates of the transitions at the seismic scale. These sharpness estimates involve the computation of scale exponents which represent order of magnitude estimates for the behavior of the wavelet coefficients.

By formulating the prestack migration in terms of the continuous wavelet transform, a possibility is created to not only reconstruct the sharpness of the reflectors but also to correct for possible anomalous AVO, induced by the scaling.

Acknowledgments

This work was supported by the Borehole Acoustics and Logging/Reservoir Delineation Consortia at the Massachusetts Institute of Technology.

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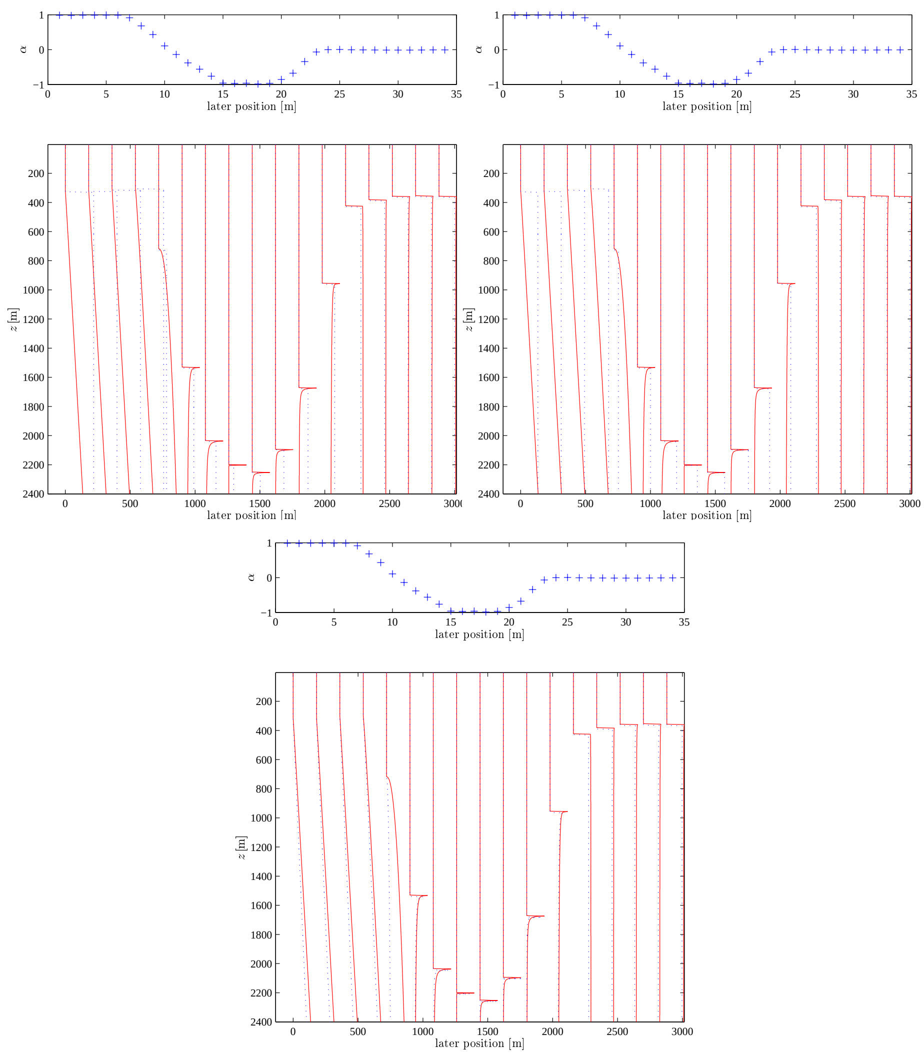

Figure 1: "Zoeppritz inversions" on data generated from models with a single transition with $\alpha \in [-1.1]$ (see the α-curves on the top of each plot). The second subplot of each plot contains the reconstructed (dashed) and original (solid lines) compressional wavespeed. Top left: reconstruction *without* renormalization; Top right: reconstruction *with* renormalization; Bottom: reconstruction *with* renormalization and sharpness information. Notice the exceeding overestimation of the c fluctuations as the sharpness (α) increases for the non-corrected inversion (top-left). This effect is known as "tuning" by a "thin" layer.