Discrimination and classification of buried unexploded ordnance using magnetometry

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Abstract

Magnetic surveys are quite successful at locating buried steel objects, including unexploded ordnance (UXO). However, many of the anomalies apparent in a magnetic image of a contaminated area are from metallic debris, shrapnel and geological variations in ferromagnetic concentration. Observations are usually made in the far-field of the object so that in most cases, we can only recover the dipole moment of a buried item. Due to self-demagnetization effects the magnitude and direction of induced magnetism varies significantly with ordnance orientation. This results in an infinite number of ordnance like objects that can reproduce a given dipole moment. To discriminate, we define a library of ordnance items expected to occur in the area and find how closely each recovered moment matches one of the UXO’s in this library. We define the size of this mismatch as the remanent magnetization and produce a prioritized dig-list on the assumption that items with lower remanence are more likely to be UXO. Such a ranking scheme proves to be very effective when implemented at two sites in Montana. The analysis reveals that live-site and emplaced UXO have significantly different remanence and implies that previous tests of magnetic discrimination performance on seeded sites have been overly pessimistic.

Index Terms

Discrimination, magnetics, unexploded ordnance, inversion, classification

I. INTRODUCTION

Unexploded ordnance pose a significant public safety hazard in many parts of the world. They occur on or near the surface and down to depths of several meters below the ground. One source of UXO’s are armed conflicts both old and more recent in regions such as Europe (World Wars I and II), the Middle-East, South-East Asia, Afghanistan, the Balkans and parts of Africa. They are also a significant problem in countries such as Canada and the United States, where they are present in areas used for military training and firing ranges. It is estimated that 15 million acres of the United States are contaminated by UXO [1], with a clean-up time-frame in the decades and a staggering cost estimate, using existing technology, in the tens to hundreds of billions of dollars [2].

The most well established techniques for ordnance detection are magnetics and electromagnetics [3]. These methods are very effective at locating buried metallic objects such as UXO. However, discriminating between intact

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UXOs and non hazardous objects such as metallic debris and shrapnel is a significant challenge [3]. Often little or no effort is invested in discrimination and, consequently, many holes are excavated for each ordnance item recovered. For example, [4] reports that only 3% (or 1,486) of the 49,521 anomalies excavated at Kaho’olawe, Hawaii, turned out to be UXO.

In this paper we investigate the potential of magnetics for both discrimination and classification. Discrimination refers to the problem of distinguishing between UXO and clutter, while classification refers to the problem of determining the correct ordnance type.

The principle of magnetic UXO detection is that buried steel ordnance cause a distortion in the Earth’s magnetic field that can be measured by a magnetometer. However, magnetic anomalies also arise from shrapnel and other ferrous debris in an area, as well as from geological variations in ferromagnetic materials. The problem is to come up with a diagnostic feature of the magnetic anomaly of a UXO that will allow it to be distinguished from these clutter items. Ideally, we would like to be able to recover the shape and size the anomaly’s source and use that information for discrimination. However, there is a fundamental ambiguity in magnetics, whereby, for instance, any magnetic anomaly can be represented by an equivalent layer of susceptibility [5]. This immediately dooms such a procedure to failure.

To proceed, we note that the response of a compact body can be decomposed into a series of moments by a multipole expansion [6]. In most cases, measurements are made in the far-field of the object (i.e. at distances several times the object’s dimensions) so that the response of the dipole component dominates due to the rapid decay with distance of the other components. Therefore, all that we can usually recover about a buried object is its dipole moment. In general, an object’s dipole moment is a consequence of both remanent and induced magnetization. Remanent magnetization is present even in the absence of an inducing field and is due to ferromagnetic domains in the steel being locked into alignment sometime during the object’s history. Induced magnetism arises because magnetic domains in a ferrous material tend to align with the direction of the ambient field.

[7] postulated that the shock of impact partially erases the remanent magnetization of a UXO. They further noted that the direction of induced magnetization in typical ordnance items is constrained to lie within about 60° of the Earth’s field. This fact was utilized by [8] for ordnance discrimination with some success. Here, we build open these earlier papers and show how remanent magnetization can be used as a diagnostic of unexploded ordnance. To discriminate, we specify a library of ordnance items expected to occur in the area and calculate a set of feasibility curves (all the dipole moments that can arise from induced magnetization alone). We then determine the minimum distance between each recovered dipole moment and the feasibility curves; this distance becomes an estimate of the object’s remanent magnetization. Due to shock demagnetization, UXO’s in the ordnance library will have low values of calculated remanence. Geological anomalies, shrapnel and other metallic debris will, in general, not match the induced model very well and hence will have large values of remanence.

Remanence therefore provides an efficient means to rank anomalies according to their UXO likelihood. To extend this method to classification, we assign each anomaly to the ordnance type that generated the lowest value of estimated remanent magnetization. As one might expect from the nature of magnetic data there is some ambiguity.
involved in this process but some success in classification can be achieved.

For the Earth’s field we use geographical coordinates; $b_\alpha = (b_{ox}, b_{oy}, b_{oz})$, i.e. $x$ is positive to the East, $y$ is positive to the North and $z$ is positive upwards. Note that this differs from the definition used to specify the International Geomagnetic Reference Field (IGRF) where $x$ is positive to the North, $y$ is positive to the East and $z$ is positive downwards.

II. ANOMALOUS MAGNETIC FIELD FROM A COMPACT FERROUS OBJECT

The magnetic field is a potential field so that it can be expressed as the gradient of a scalar potential, $b(x_\alpha) = \nabla \phi(x_\alpha)$. Using the geometry of Figure 1, the scalar potential of a compact body occupying the volume $V$ with magnetization $M(x)$ is given by the expression

$$\phi(x_\alpha) = \frac{1}{4\pi} \int_V M(x) \cdot \nabla \left(\frac{1}{r}\right) dx$$

where $r = ||r||$ is the distance between the source and observation point. A well known and general method [6] for calculating the magnetic field via this equation is by the multipole method. This proceeds by expanding $\frac{1}{r}$ as a Taylor series about the origin. The magnetic field is then expressed as a sum of moments. The first term in the expansion is the dipole moment which is a 3 element vector whose $i$-th component can be shown to be equal to

$$m_i = \int_V M \cdot \nabla x_i dx = \int_V M_i dx$$

When the magnetization vector is constant the dipole moment is the product of the magnetization with the volume,

$$m = M \int_V dx = MV$$

The contribution of the dipole to the magnetic field decays as the 3rd power of distance from the object and dominates the far-field,

$$b(x_\alpha) = \mu_0 \frac{3}{4\pi r_o^3} \left(3 \frac{r_o \cdot m}{r_o^3} - m\right)$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ is the permeability of free space and $r_o = ||r_o||$ is the distance between the center of the object and the source. The next moment is the quadrupole which is a rank 2 tensor with components given by the expression

$$q_{ij} = \int_V M \cdot \nabla (x_i x_j) dx = \int_V (M_i x_j + M_j x_i) dx$$

It decays as the 4th power of distance. For any compact body with 3-axes of symmetry (such as a spheroid or cylinder) one can show from the above equation that there will be no quadrupole contributions. A non-zero quadrupole requires asymmetry in either the geometry or the magnetic field.

The last moment we consider is the octupole, a rank 3 tensor

$$o_{ijk} = \int_V M \cdot \nabla (x_i x_j x_k) dx$$

The octupole dies out as the 5th power of distance so that its contribution decays very rapidly. The octupole moment will generally be non-zero even for symmetric bodies such as cylinders and spheroids (although note that a sphere
has a zero octupole moment). The main effect of symmetry is to reduce the number of independent components of the octupole tensor.

The magnetization of the buried ferrous item arises from both remanent \( M_{\text{rem}} \) and induced means \( M_{\text{in}} \), so that the total magnetization \( M \) is then

\[
M = M_{\text{rem}} + M_{\text{in}}
\]

Remanent magnetism is present even in the absence of an inducing field and is due to ferromagnetic domains being locked into alignment at some stage in the history of the object. Induced magnetism arises because magnetic domains in a ferrous material tend to align with the direction of the Earth’s field. The ease with which the moments align, and hence the strength of the magnetization, depends on the magnetic susceptibility of the body, \( \chi \) (which we assume is constant). For low susceptibilities (\( \chi < 0.1 \)), the induced magnetization and applied field \( b_o \) are parallel and related by

\[
M_{\text{in}} = \frac{\chi}{\mu_o} b_o
\]

Self-demagnetization effects become important for a compact body with large susceptibility (such as steel where \( \chi \sim 500 \)). Self-demagnetization refers to the extent that the induced field is reduced due to the shape of the object. It arises due to the boundary conditions that must be satisfied across a discontinuous change in magnetic susceptibility (normal component of magnetic induction and tangential component of magnetic field strength are continuous) and can result in both a reduction and a change in direction of the induced magnetization.

The effects of demagnetization can be expressed as an effective susceptibility. For an arbitrarily shaped body, the effective susceptibility will be a rank two tensor, \( \chi(x, \chi, \Omega) \), that is a function of position \( x \), the susceptibility \( \chi \) and shape of the body, \( \Omega \). At each point in the body, the induced magnetization will be

\[
M_{\text{in}}(x) = \frac{\chi(x, \chi, \Omega) b_o}{\mu_o}
\]

From Equation (4), the induced dipole moment will then be,

\[
m_{\text{in}} = \mu_o^{-1} \left[ \int_V \chi(x, \chi, \Omega) \, dx \right] b_o = \frac{V \bar{\chi} b_o}{\mu_o},
\]

where \( \bar{\chi} \) is the average effective susceptibility within the body,

\[
\bar{\chi} = \frac{1}{V} \int_V \chi(x, \chi, \Omega) \, dx.
\]

We note that equivalent expressions can be derived for the quadrupole and octupole moments using Equations (5) and (6) as starting points.

Now, \( \bar{\chi} \), is a \( 3 \times 3 \) matrix that encapsulates the integrated effect of self-demagnetization. It will always be possible to diagonalize this matrix and find a set of three principal axes so that \( \bar{\chi} = A^T \text{diag}(\bar{\chi}_1, \bar{\chi}_2, \bar{\chi}_3) A = A^T \bar{\chi}_d A \), where \( A \) is an orthogonal matrix and \( \bar{\chi}_1, \bar{\chi}_2 \) and \( \bar{\chi}_3 \) are the average effective susceptibilities along the principal axes. The equation for the induced dipole moment then becomes

\[
m_{\text{in}} = \frac{V}{\mu_o} A^T \bar{\chi}_d A b_o.
\]
For the most part, unexploded ordnance are bodies of revolution so that $\bar{\chi}_1 = \bar{\chi}_2$ (fins and other asymmetric structures are typically made of aluminum and therefore are not ferromagnetic). If we adopt the convention that $\phi$ is the angle clockwise from North of the projection of the z-axis ($\bar{\chi}_3$-axis) of the body onto a horizontal plane, while $\theta$ is the dip angle (positive upwards) of the z-axis relative to that plane, then the orthogonal matrix becomes

$$
A = \begin{bmatrix}
\cos \phi & -\sin \phi & 0 \\
\sin \theta \sin \phi & \sin \theta \cos \phi & \cos \theta \\
\cos \theta \sin \phi & \cos \theta \cos \phi & -\sin \theta 
\end{bmatrix}.
$$

(13)

The geometric interpretation of $A$ (the so-called Euler rotation tensor) is that it rotates a field in geographic coordinates to body centered coordinates while its transpose $A^T$ does the reverse.

To calculate the dipole moment induced in a given body of revolution, we need to know its orientation ($\theta$ and $\phi$), volume $V$, and the average effective susceptibilities ($\bar{\chi}_2$ and $\bar{\chi}_3$) along the two principal axes. Unfortunately, except for simple shapes such as spheroids, analytical determination of $\bar{\chi}_2$ and $\bar{\chi}_3$ is not possible.

A. The magnetic field of a prolate spheroid

Solid prolate spheroids are reasonable approximations to the shapes of many ordnance items and their effective susceptibility can be analytically calculated. Consequently they have found extensive use in the magnetic modelling of UXO’s [9], [7]. Close agreement between observed anomalies over test stands and spheroid fits have been demonstrated by [9] and [3]. Furthermore, the magnetic anomaly from a solid spheroid has been shown to be very similar to a hollow spheroid [7]. Therefore, for the present moment we will use solid prolate spheroids for our modelling and will comment on the use of other shapes later in the paper.

We assume the spheroid has a diameter $a$, length $L = ae$ (where $e > 1$ is the aspect ratio) and a magnetic susceptibility, $\chi$. A solution of a boundary value problem [6] shows that the effective susceptibility is constant throughout the body and given by the expression

$$
\bar{\chi}_i = \frac{\chi}{1 + \alpha_i \chi/2}
$$

(14)

where $\alpha_1 = \alpha_2$ and $\alpha_3$ are self-demagnetization factors that are dependent on the aspect ratio, $e$ (see Equation (18) in the Appendix). Once $\chi$ exceeds a few hundred, the induced magnetization for a spheroid becomes virtually independent of susceptibility. This can easily be seen from Equation (14) with large $\chi$ because then $\bar{\chi}_i \approx 2/\alpha_i$.

As steel typically has susceptibilities of several hundred this allows $\chi$ to be eliminated from our calculations. For $e = 4$ (typical of certain types of UXO) and with $\chi \to \infty$ the demagnetization factors are $\bar{\chi}_1 = \bar{\chi}_2 = 2.2$ and $\bar{\chi}_3 = 13.1$, so that the strength of the induced magnetization is around six-times larger when the field is aligned with the spheroid’s semi-major axis compared to when it is perpendicular.

As a spheroid is symmetric and the induced magnetization is constant, one can see from inspection of Equation (5) that the quadrupole moment will be zero. Furthermore, symmetry reduces the number of independent components of the octupole down to 6. The specification of the octupole term is quite complicated, and we refer the interested reader to [9] for more details.
The rapid decay of the octupole term with distance (noted earlier) means that once the sensor distance exceeds a few body lengths, the field is essentially dipolar. Considering that real magnetic profiles are contaminated by noise, observing the octupole response in the presence of the dipole response will generally be very difficult. It will only be possible when the sensor is very close to the ordnance, or when the noise levels are low and the spatial positioning extremely accurate. However, even in this situation the spheroid octupole may not provide useful information. The main reason is that the octupole encapsulates detailed information on the shape of the body whereas a prolate spheroid is not an accurate approximation of the shape of a real UXO. It is, however, sufficiently accurate for the bulk information on shape and volume provided by the dipole.

### III. Discrimination and Classification Using Magnetometry

We now turn to the issue of discriminating UXO’s from other non-hazardous items such as shrapnel, metallic debris and geology. It is well known that inversion of magnetic data has a fundamental ambiguity whereby any anomaly can be reproduced by an equivalent layer of susceptibility [5]. At this time, we restrict our attention to spheroids and assume that the anomalous field can be sufficiently described by the dipole moment alone.

The dipole moment of a sphere is constrained to lie in the direction of the applied field. On the other hand, the angle the induced dipole moment of a spheroid makes with the Earth’s field will vary with its orientation. When the semi-major axis is at an angle \( \theta \) relative to the applied field, one can show that the deviation angle between the induced magnetization and applied field will be

\[
\psi(\theta) = \arccos \left( \frac{\bar{\chi}_2 \sin^2 \theta + \bar{\chi}_3 \cos^2 \theta}{\sqrt{\bar{\chi}_2^2 \sin^2 \theta + \bar{\chi}_3^2 \cos^2 \theta}} \right).
\]

The maximum deviation angle (between the induced and applied fields) is bounded and dependent on the shape of the spheroid [7]. For ordnance items up to an aspect ratio of 7, the deviation angle never exceeds 55° (in the absence of remanent magnetism). This fact was utilized by [8] for ordnance discrimination with some success. [7] postulated that the reason the method works is the shock-demagnetization that occurs when an ordnance hits the ground but does not explode. The force of impact is sufficient to cause magnetic domains within the ordnance to become randomly aligned, thus erasing any remanent magnetization. Consequently ordnance should not usually have a significant remanent magnetization. On the other hand, when the ordnance explodes, the shrapnel is heated and deformed so that it may acquire permanent magnetism.

The deviation angle tells only part of the story and ignores useful information on the magnitude of the dipole. We assume for the moment that there is no remanent magnetization and we will attempt to identify ordnance items from their dipolar field. For a given spheroid, the induced dipole moment will be dependent on the angle \( \theta \) that the spheroid axis makes with the Earth’s field. There is no azimuthal dependence due to the spheroid’s symmetry. As the orientation \( \theta \) varies the dipole moment will vary as

\[
m(\theta) = \frac{V\|b_{\text{el}}\|}{\mu_0} \sqrt{\bar{\chi}_2^2 \sin^2 \theta + \bar{\chi}_3^2 \cos^2 \theta}
\]
while the deviation angle will change according to Equation (15). Figure 2 shows these curves as polar plots for four different ordnance items: 76 mm projectile, 81 mm mortar and 105 and 155 mm projectiles. Fitting a dipole to observed data will produce a single point in this polar plot that needs to be matched to an ordnance item. The curves for the 76 mm projectile and the 81 mm mortar are very similar, indicating that discriminating between these items may be difficult. Further, certain orientations lead to identical dipole moments for different ordnance items.

The analysis in the last paragraph indicates that there can be ambiguity in classification using magnetometry even after assuming the source is a spheroid. We show in the Appendix that for a given spheroid at a particular orientation, there are an infinite number of other spheroids that could have produced the same dipole moment. For example, Figure 3 and Table I show the family of spheroids that can produce the same dipole moment as a 105 mm projectile orientated at 45° to the Earth’s field (magnitude: 0.6 Am²; angle: 34.4° relative to the Earth’s field). Aspect ratios greater than unity correspond to prolate spheroid, while less than unity they correspond to oblate spheroids. For many aspect ratios there are two diameters that can reproduce the moment. There are two critical points (0.31 and 2.77) where only one diameter can be used and there is a range of aspect ratios (0.32 to 2.76) where no orientation of the spheroid can reproduce the moment. The reason the ambiguity occurs is that we can compensate for a change in aspect ratio by changing the diameter and orientation of the spheroid (Figure 3b). When two diameters are allowed for a given aspect ratio, the larger diameter involves rotating the spheroid away from the Earth’s field, while the smaller involves a rotation towards the Earth’s field. In summary, inability to constrain the orientation causes ambiguity.

Our method for both discrimination and classification in the presence of both ambiguity and non-zero remanent magnetization is as follows. For each of the $N_o$ ordnance items in the library, generate the feasibility curves, $\{m_k(\theta), k = 1, \ldots, N_o\}$, as in Figure 2. For a particular recovered dipole moment $m$, we calculate the minimum distance between it and each of the dipole curves

$$\Delta m_k = \min_{0 \leq \theta < \pi} \|m - m_k(\theta)\|$$

We assume that any discrepancy is due to remanent magnetism which, as a percentage of the recovered moment, will be $\gamma_k = 100\Delta m_k/\|m\|$. For example, in Table II we show the remanent magnetizations for items 1 and 2 in Figure 2. For each item, we assume that the most likely source of the anomaly is the UXO with the lowest percentage remanent magnetism. Thus item 1 is most likely a 155 mm projectile, while item 2 is most likely a 105 mm projectile. Clearly, such an classification procedure will be imperfect and will become increasingly difficult as the number of items in the library increases and/or as the remanent magnetization increases.

To discriminate between UXO and non-UXO we rank items according to the remanent magnetization required to match the best fitting UXO in the library. Thus item 1 with 22% remanence required to match a 155 mm projectile is more likely a UXO than item 2 with 40% remanence required to match a 105 mm projectile. Due to shock demagnetization on impact, unexploded ordnance should have a small remanent magnetization and will therefore have a small predicted value of remanence (if the ordnance is included in the library). Shrapnel and metallic debris
will be more likely to be remanently magnetized and will also not fit any of the feasibility curves very well. Consequently, they should have a large value of remanence.

Note that the polar plot in Figure 2 is symmetric about the horizontal axis, so that the top half of the curve alone would be sufficient. However, we utilize both halves of the curve by adopting the convention that moments with azimuths to the East of magnetic North are negative and those to the West are positive.

IV. APPLICATION TO LIVE-SITES

The previous section demonstrated that, in many situations, it would be difficult to recover more than the dipole component of a spheroid’s magnetic anomaly. Therefore, rather than attempting a direct inversion for the spheroid location and dimension, we apply a two-step procedure:

1) Invert for the best-fitting dipole moment and location using an interior-reflective Newton method [10]; and
2) Use the recovered dipole moment for discrimination and ordnance classification.

A. The Guthrie Road, Montana Dataset

Guthrie Road in the Helena Valley, Montana was used by the Montana Army National Guard (MTARNG) in the 1950s for military training. Archival search and preliminary surface clearance revealed that only 76 mm projectiles and 81 mm mortars were fired into this area.

In the summer of 1998, Geophysical Technology Limited (GTL) of Brisbane, Australia, conducted a magnetometer survey of the area using an all-terrain vehicle towing an array of eight cesium vapor magnetometers [11]. The system was equipped with a real-time global positioning system that allowed anomalies to be relocated to within about 20 centimeters.

At Guthrie Road, 840 anomalies were tagged as potential UXO, and these were excavated in the summer of 1999. We analyzed 804 of the anomalies (we either didn’t have the data for the remaining items or the dig-sheet validations were incomplete). Of the 804 anomalies, 724 had reliable dipole fits while 80 had fits that could not be relied upon (see Table III). There were a total of 80 UXO’s consisting of thirty-three 76 mm projectiles and forth-seven 81 mm mortars. Of these 80 UXO’s, 56 were live while the other 28 were emplaced. That is, they are items that were buried by the Montana Army National Guard specifically as a quality assurance check of the effectiveness of the geophysical survey. These emplaced ordnance items were sourced from other parts of the survey area.

In Figure 4a we plot the feasibility curves for 76 mm projectiles and 81 mm mortars along with the recovered dipole moments for all items validated as either 76 or 81 mm caliber UXO. The recovered moments for the 76 mm projectiles tend to lie fairly close to the 76 mm feasibility curve, while those for the 81 mm lie close to the 81 mm feasibility curve. The recovered moments of the UXO generally lie within a wedge of ±60° relative to the Earth’s field although there are several moments with larger angles. In contrast, the recovered moments for shrapnel, metallic debris and geology (Figure 4b) have a wide distribution of angles.
The results of discriminating using the deviation angle alone are shown in Figure 5a. Up to an angle of about 30° there is a clear separation between UXO and non-UXO with around 75% of UXO having angles smaller than 30° compared to less than 20% of non-UXO. All but two of the remaining UXO lie between 30° and 60°. However, around 70% of shrapnel and 35% of geology and metallic debris have angles less than 60°, thus the efficiency of the discrimination method is significantly diminished.

Discrimination using the percentage of remanent magnetization turns out to be much more efficient (Figure 5b). Around 80% of live UXO and 70% of emplaced UXO have a remanence of 20% or less compared to less than 10% of shrapnel, metallic debris and geology. In addition, all detected UXO are recovered with a remanence of 50% or less, compared to just over 20% of non-UXO. Thus remanence appears to provide a very useful method for discrimination.

The cumulative distributions with respect to both angle and remanent magnetization were different for live UXO compared to emplaced UXO (Figure 5). Except for a few difficult outliers, the remanence of the live-site UXO were generally less than the emplaced UXO. This difference is statistically significant at a 99% level according to a Kolmogorov-Smirnov test [12]. The fact that live-site and emplaced UXO have different remanent properties has interesting implications for both discrimination and for testing of discrimination methodologies at seeded sites. We will defer further discussion on this point until after we present the results on a different site.

Receiver operating characteristics of the method are shown in Figure 6. Defining the False-Alarm-Rate (FAR) as the number of non UXO’s excavated per UXO the figure reveals that around 90% of UXO are recovered at a FAR equal to one (that is one non-UXO dug per UXO). The remaining 10% of UXO are harder to recover but nevertheless, the last UXO occurs when the FAR is less than two. This means, that, on average, for every three holes dug there will be one UXO. Without discrimination, one UXO was recovered in every eight holes at Guthrie Road.

Regarding ordnance identification, the remanent magnetization is smaller for the correct UXO, 70% of the time. Given that there were only two items in the ordnance library the classification task was relatively simple. In addition, there was a fairly consistent difference in the magnitude of the moments for the two ordnance items, so that we could do about as well by thresholding on magnitude alone.

Finally in Figure 7 we show the one-sigma and two-sigma confidence regions on the recovered moments for items validated as 76 mm projectiles and 81 mm mortars. These were calculated using standard least-squares error analysis techniques [13]. Many of the confidence regions are quite small indicating that the recovered moments are reliable. However, there are a significant number of anomalies with relatively large confidence regions. This implies that, for certain items, uncertainty in the recovered dipole moments is likely to make correct classification of ordnance difficult.

B. The Limestone Hills, Montana Dataset

The second site that we consider is at Limestone Hills which is also in Montana but lies in an area with different geology, landforms and military usage. Previous surface sweeps and archival search revealed five different caliber
projectiles had been used; 76 mm AP/T (Armor Piercing Tank), WP (White Phosphorous) and HE (High Explosive), 90 mm AP/T, WP and HE, 105 mm illumination, WP and HE, 4.2 illumination and HE, and 155 mm illumination, HE and WP. The size variability of the different types for a given caliber are quite small so that we use a single set of dimensions for each caliber.

The data were collected by Geophysical Technology Limited in 1999 and 2000 with a hand-held quad-sensor magnetometer with a cotton thread odometer used for along line positioning. Data positioning was less accurate than at Guthrie Road, so that there is a greater uncertainty in the recovered dipole moments. Of the 360 validated anomalies at Limestone Hills, 318 could be reliably modelled with a dipole (Table III); 22 of these were from live UXO and 43 were from emplaced UXO.

A polar plot of the recovered dipole moments of the validated UXO along with the feasibility curves from the ordnance library are shown in Figure 8a. The live-site UXO tend to cluster relatively close to the feasibility curves while the emplaced UXO have a greater spread of recovered moments. The moments of several emplaced UXO have angles exceeding $60^\circ$ (in one case the angle exceeds $90^\circ$). The moments of shrapnel, metallic debris and geology have a broad spread of angles, but tend to have quite small magnitudes (Figure 8b).

If we attempt to discriminate on the basis of angle alone (Figure 9a), we can do a reasonable job for live-UXO (100% recovery of detected UXO with under 50% of non-UXO mistakenly classified as UXO) but do a very poor job for emplaced UXO. By the time the last emplaced UXO turns up with an angle just over $90^\circ$ almost 90% of shrapnel, metallic debris and geology have been classed as UXO. Using remanent magnetization as a discriminant we are able to do much better (Figure 9b). All live UXO are recovered with less than 45% remanence at which point less than 15% of non-UXO have been misclassified. The emplaced UXO turn out to be more difficult to correctly classify with two outliers requiring over 100% remanence. Nevertheless, even at this large value of remanence, only 35% of false alarms have been misclassified. The difference in the remanent properties of live site and emplaced UXO are even more apparent here than they were at Guthrie Road.

The receiver-operating-characteristics for the remanence discriminant, reveal a FAR of around 1 for 95% of all UXO (live and emplaced) and around 90% for live UXO (Figure 6). The reason that live-site UXO have a higher FAR, even though they tend to have lower remanence, is that there are only 22 live UXO out of 65 total UXO. This low number of live-site UXO’s distorts the ROC curve. All detected UXO are correctly classified with a FAR of less than 1.5, which means that on average 2 UXO are found in every 5 holes (compared to 1 UXO in every 5 holes without discrimination).

Of 63 UXO at Limestone Hills, 16 were correctly classified (this compares to the mean value of 10.5 for random chance alone), 35 were either classified correctly or as the next highest or lowest caliber projectile (compared to a mean value of 28 for random chance), while 55 were classified correctly or as the next two highest or lowest caliber projectiles (compared to a mean value of 42 for random chance). Thus there is only a marginal ability to classify the ordnance into the correct caliber class.
V. DISCUSSION

The results presented in this paper demonstrate that remanent magnetization provides a very powerful discriminant of UXO likelihood. At both the Guthrie Road and Limestone Hills sites, prioritizing the dig-list according to remanence allowed 100% of detected UXO to be recovered while leaving a significant proportion of anomalies due to shrapnel, metallic debris and geology in the ground. At both sites, the false-alarm-rates for 90% recovery of detected UXO were less than one. The last 10% of detected UXO were harder to distinguish from non-UXO but nevertheless the FARs for 100% recovery at both sites were less than two. This is a significant improvement on the FAR at both sites without discrimination (8 at Guthrie Road and 4 at Limestone Hills).

An issue when applying the methodology prior to excavation is that the number of detected UXO are unknown. That is, we can’t tell a-priori when to stop digging. Our suggested methodology is to form an initial dig-list comprising all those items with a remanence less than 50% plus a random selection of anomalies with larger remanence. One may subsequently need to augment this dig-list with items further down the ranking list until a certain number of predefined holes (e.g. 50) have been dug without any further UXO being recovered. The number of predefined holes could be determined by regulators and stakeholders.

While magnetic discrimination appears to be viable, classification of potential UXO into categories is more difficult. The poor performance of the classification methodology is a consequence of a number of factors;

1) Uncertainty: The recovered dipole moments are subject to uncertainty, with the size of this uncertainty often sufficient to change the classification. The results of any advanced classification method will always be limited by the quality of the data. If noise levels are high or spatial positioning inaccurate the data will not be able to support quantitative analysis. Additionally, even if the data are of high quality, the recovered dipole model may still be inaccurate if there are higher order moments present in the data. In that case, one can implement an inversion routine that accounts for higher order moments [14].

2) Ambiguity: Many differently shaped ordnance like objects can reproduce a given dipole moment. This ambiguity is caused by the absence of information on the higher order moments in the data.

3) Modelling error: The trajectories of the induced magnetization curves are approximate. In particular, we use an equivalent sized spheroid to generate the curves used in this paper. There are two possible methods to improve on these curves. The first is to model the response of realistically shaped ordnance and from there obtain the average effective susceptibilities required in Equations (15) and (16). The second is to measure the magnetic field on a test-stand above a UXO which is swept through a range of orientations. By recording the full 3-D orientation of the item, any remanent magnetization that the item possesses can be characterized. Furthermore, by inverting for a dipole moment for each orientation one can then empirically determine the best fitting set of effective susceptibilities.

4) Remanence: Unexploded ordnance are likely to have some remanent magnetization. The problem with remanent magnetization is that it is not diagnostic of ordnance shape and size, and serves to complicate the classification problem.
The issue of remanence is, in fact, critical to the performance of the discrimination method. For items that have not been shock demagnetized (such as grenades) the recovered moment may not agree very well with the induced model. Thus the item may have a large calculated remanence and hence be ranked quite low in the discrimination ranking list. For items that have not been shock demagnetized we would not recommend using the remanence discrimination method. In that case, the orientation of the dipole moment will provide little useful information and one would be forced to discriminate on dipole magnitude alone.

The results presented in this paper clearly demonstrated that emplaced UXO can have significantly larger remanent magnetization than live-site UXO. There are several possible explanations for this discrepancy:

1) Shock remagnetization whereby the UXO acquires a remanent component aligned with the induced field during impact and after the initial remanence has been erased by shock demagnetization. When the ordnance are moved and then re-buried, the orientation will inevitably change so that the relation between the remanent and induced components of the item will change.

2) The shock experienced by the UXO when moved may cause it to acquire a remanent magnetization.

3) Viscous remanent magnetization which is a remanent magnetization the UXO acquires over time. Gradually, more and more magnetic domains gain sufficient thermal energy to overcome internal barriers and flip their magnetization to an angle more in alignment with the external field. When the object is moved and reburied not enough time elapses for the viscous remanence to reestablish itself in the new field direction.

Clearly, additional research into the issue of remanence is required if the method is to prove viable for magnetic discrimination for UXO clearance. The fact that live-site UXO have lower remanence than emplaced UXO exposes the potential pitfalls of testing magnetic discrimination on seeded sites. If the emplaced UXO are not completely demagnetized then the discrimination method will perform worse than it should. Conversely, if the emplaced UXO were completely demagnetized then the discrimination method may perform better than it should (because real UXO may have some remanence). From previous analysis [15] we know that the former is more likely, implying that tests of magnetic discrimination at live-sites have disadvantaged magnetics in the past. These observations imply that adequate tests of magnetic discrimination performance can really only be conducted on live sites.

VI. CONCLUSION

Magnetic remanence provides a very effective means for discrimination of unexploded ordnance that have been shock demagnetized. In the case-studies presented here, all UXO could be recovered with less than 20% of other items incorrectly declared UXO. As observations are usually made in the far-field of the object, the most information that can usually be recovered is the dipole moment. There is an ambiguity involved in determining the object shape and size from the dipole moment alone. Coupled with errors in recovering the dipole moment, remanent magnetization and modelling deficiencies this means that classification of UXO into different caliber projectiles is difficult. Lastly, the fact that the remanent properties of live-site and emplaced UXO are different may have disadvantaged tests of magnetic discrimination at seeded sites in the past.
APPENDIX I

AMBIGUITY IN THE DIPOLE SOLUTION

Prolate spheroids of many different diameters and aspect-ratios can reproduce a given observed dipole by appropriate rotation of the body relative to the Earth’s field. Equation 14 expressed the average effective susceptibilities, \( \bar{\chi}_2 \) and \( \bar{\chi}_3 \), in terms of the self-demagnetization factors, \( \alpha_1 \) and \( \alpha_2 \), that depend only on the aspect ratio of the spheroid,

\[
\alpha_1 = \alpha_2 = \frac{e(e + E)}{e^2 - 1} \quad \text{and} \quad \alpha_3 = \frac{-2e(e^{-1} + E)}{e^2 - 1}
\]

with

\[
E = \log\left(\frac{e - \sqrt{e^2 - 1}}{\sqrt{e^2 - 1}}\right)
\]

for a prolate spheroid \((e > 1)\) and

\[
E = \frac{\arctan(e\sqrt{e^2 - 1} - \pi/2)}{\sqrt{1 - e^2}}
\]

for an oblate spheroid \((e < 1)\). For the special case of a sphere \((e = 1)\) then

\[
\alpha_1 = \alpha_2 = \alpha_3 = \frac{2}{3}
\]

To characterize the spheroid ambiguity we chose our coordinate system so that the z-axis is aligned with the Earth’s field,

\[
b_\alpha = (0, 0, b_o)^T
\]

We want to determine if the dipole \( \hat{m} \) from a particular spheroid (diameter \( a \) and aspect-ratio \( e \)) can match an observed dipole moment \( m \). Now the induced dipole for the spheroid is given by the following equation

\[
\hat{m} = VA^T \bar{\chi} A b_\alpha
\]

Expanding this equation we find

\[
\hat{m} = \frac{b_o \pi e a^3}{6 \mu_o} \begin{bmatrix}
  (\bar{\chi}_2 - \bar{\chi}_3) \cos \theta \sin \theta \sin \phi \\
  (\bar{\chi}_2 - \bar{\chi}_3) \cos \theta \sin \theta \cos \phi \\
  \bar{\chi}_2 \cos^2 \theta + \bar{\chi}_3 \sin^2 \theta
\end{bmatrix}
\]

Now we want to find \( a, e, \theta \) and \( \phi \) so that \( \hat{m} = m \). We can find an expression for the spheroid diameter by equating the norms of the two dipole moments, resulting in

\[
a^3 = \frac{6 \mu_o ||m||}{b_o \pi e \sqrt{\bar{\chi}_2 \cos^2 \theta + \bar{\chi}_3 \sin^2 \theta}}
\]

For \( \phi \) we use,

\[
\frac{m_1}{m_2} = \frac{\hat{m}_1}{\hat{m}_2} = \frac{\sin \phi}{\cos \phi} = \tan \phi
\]

which leads to

\[
\phi = \arctan\left(\frac{m_1}{m_2}\right)
\]

This implies that the spheroid has the same azimuth as the dipole moment.
Determination of $\theta$ is a little more difficult. We start by using the second row of Equation (24) and substitute in Equation (25) for the diameter,

$$\frac{m_2}{\cos \phi ||m||} = \frac{(\bar{\chi}_2 - \bar{\chi}_3) \cos \theta \sin \theta}{\sqrt{\bar{\chi}_2^2 \cos^2 \theta + \bar{\chi}_3^2 \sin^2 \theta}}. \quad (28)$$

Note that if $\cos \phi = 0$ we would use the first row of Equation (24) to derive an equation in terms of $m_1$ and $\sin \phi$. Let the LHS of Equation (28) be $\omega$, expand out the square-root on the denominator of the RHS, and use the substitution

$$\sin^2 \theta = 1 - \cos^2 \theta. \quad (29)$$

After some algebraic manipulation Equation (28) reduces to

$$((\bar{\chi}_2 - \bar{\chi}_3)^2 \cos^4 \theta \left[\omega^2 (\bar{\chi}_2^2 - \bar{\chi}_3^2) - (\bar{\chi}_2 - \bar{\chi}_3)^2\right] \cos^2 \theta + \omega^2 \bar{\chi}_3^2. \quad (30)$$

This is a quadratic equation in $\cos^2 \theta$ which has a real solution if

$$\Delta = \hat{b}^2 - 4\hat{a}\hat{c} \geq 0, \quad (31)$$

where

$$\hat{a} = (\bar{\chi}_2 - \bar{\chi}_3)^2 \quad (32)$$

$$\hat{b} = \omega^2 (\bar{\chi}_2^2 - \bar{\chi}_3^2) - (\bar{\chi}_2 - \bar{\chi}_3)^2 \quad (33)$$

$$\hat{c} = \omega^2 \bar{\chi}_3^2. \quad (34)$$

The solution in terms of $\theta$ is then

$$\theta = \pm \arccos \left(\sqrt{-\frac{\hat{b} \pm \sqrt{\Delta}}{2\hat{a}}} \right). \quad (35)$$

The $\pm$ terms in the above equation emphasize that when $\Delta > 0$ there are actually two possible solutions.

Determination of which spheroids can reproduce the observed dipole proceeds as follows. Firstly, we chose an aspect-ratio $e$ and calculate the effective susceptibilities using Equations (14) and (18). We then substitute Equations (32) to (34) into Equation (31) to determine if a solution can be found for this aspect-ratio. If it can, we calculate $\theta$ by Equation (35), and finally we calculate the spheroid diameters (usually there will be two) by Equation (25).

ACKNOWLEDGMENT

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REFERENCES


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Dr Billings received a BSc (Hon) in Theoretical Physics from The Australian National University in 1992 and a PhD in Agriculture at the University of Sydney with the thesis: Geophysical aspects of soil mapping using airborne gamma-ray spectrometry. Since completing his PhD he has been involved in contract and consulting work with Rio Tinto Exploration, Geophysical Technology Limited, Desmond Fitzgerald and Associates and both CSIRO Land and Water and CSIRO Earth Observation Center. Since November 2000 he has been a Killiam Post Doctoral Fellow at The University of British Columbia, where he is working on methods for locating and identifying unexploded ordnance.
<table>
<thead>
<tr>
<th>Label</th>
<th>Diameter (mm)</th>
<th>Aspect ratio</th>
<th>Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>82</td>
<td>5</td>
<td>40.5</td>
</tr>
<tr>
<td>B</td>
<td>138</td>
<td>5</td>
<td>84.0</td>
</tr>
<tr>
<td>C</td>
<td>138</td>
<td>2.82</td>
<td>57.2</td>
</tr>
<tr>
<td>D</td>
<td>327</td>
<td>0.06</td>
<td>53.4</td>
</tr>
<tr>
<td>E</td>
<td>327</td>
<td>0.31</td>
<td>33.4</td>
</tr>
</tbody>
</table>

TABLE I: Spheroid dimensions, and their angles relative to the Earth’s field, that produce the same dipole moment as a 105 mm projectile at 45° inclination. The label refers to Figure 3

<table>
<thead>
<tr>
<th>Remanence</th>
<th>Item 1</th>
<th>Item 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>76 mm projectile</td>
<td>60%</td>
<td>78%</td>
</tr>
<tr>
<td>81 mm mortar</td>
<td>26%</td>
<td>62%</td>
</tr>
<tr>
<td>105 mm projectile</td>
<td>28%</td>
<td>40%</td>
</tr>
<tr>
<td>155 mm projectile</td>
<td>22%</td>
<td>65%</td>
</tr>
</tbody>
</table>

TABLE II: Remanent magnetization required for item 1 and 2 in Figure 2 to match each UXO’s in the library.

<table>
<thead>
<tr>
<th>Item</th>
<th>Guthrie Road</th>
<th>Limestone Hills</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good fits</td>
<td>Failed fits</td>
</tr>
<tr>
<td>Live UXO</td>
<td>52</td>
<td>4</td>
</tr>
<tr>
<td>Emplaced UXO</td>
<td>28</td>
<td>0</td>
</tr>
<tr>
<td>Shrapnel</td>
<td>223</td>
<td>18</td>
</tr>
<tr>
<td>Metallic debris</td>
<td>312</td>
<td>25</td>
</tr>
<tr>
<td>Geology</td>
<td>109</td>
<td>33</td>
</tr>
<tr>
<td>Total</td>
<td>724</td>
<td>80</td>
</tr>
</tbody>
</table>

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Fig. 2:
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Fig. 4:

(a) Items validated as UXO

(b) Non-UXO
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(a) Items validated as 76 mm

(b) Items validated as 81 mm

Fig. 7:
Fig. 8:

(a) Items validated as UXO

(b) Non-UXO
Fig. 9:

(a) Deviation angle

(b) Remanence